where

$$C_0 = \frac{\gamma_i^{FCA} N_0}{V_{eff}} \frac{|R|^2}{\Omega^2 \mathrm{Im}(R)\sqrt{1+\Omega^2 \tau_{fc}^2}} \tag{35}$$

C_0 depends only on N_0 and Ω , which are easily calculated from the steady-state solutions and the eigenvalues of the linearized system (see above: solving the equations: steady state and small perturbations).

For oscillations triggered by an input optical power higher than 2 mW, the expressions of Ω and R reduce to $\Omega \sim \frac{\gamma_i^{FCA} N_0}{V_{eff}} - \triangle\omega$ and $R \sim i\Omega$, and the expression of C_0 simplify to

$$C_0 \sim \frac{\gamma_i^{FCA} N_0}{\tau_{fc}\Omega^2 V_{eff}} \sim \frac{1}{\Omega\tau_{fc}} \times \left(1 + \frac{\triangle\omega}{\Omega}\right) \tag{36}$$

C_0 will be lower at high frequencies, i.e. the signal will be more sinusoidal.

The same method can be used if the two-photon absorption and the free carrier absorption are taken into account in the calculation of Eq. (24). The expressions of E_0, E_1, E_2 and C_0 then become more complicated but they still simplify for an input optical power higher than 2 mW and we get for C_0 :

$$C_0 \sim \frac{1}{\Omega\tau_{fc}} \times \left(\left(1 + \frac{\triangle\omega}{\Omega}\right) - \frac{3\gamma^{TPA}\tau_{fc}E_0}{4\hbar\omega}\right) \tag{37}$$

Numerically, for a detuning of -20 pm, C_0 varies between 0.27 and 0.084 for an input power varying from 1.5 mW to 20 mW. For P_{in}= 2 mW, $C_0 = 0.24$ and $E_1/E_0 = -21.4$ dB, which gives us $E_2/E_1 \simeq -33.6$ dB, which is very close to the values found by the numerical simulation (see Fig. 4b) : $E_1/E_0 = -21.3$ dB and $E_2/E_1 = -34.7$ dB. The amplitude of the second harmonic is very low compared to the fundamental, which justifies the approximation of the energy function by a sinusoid we made at the beginning.

In the case of a microring resonator of the kind described in Ref. [16], we have according to our simulations $\alpha^{TPA} = 0.042$ and $\alpha^{FCA} = 0.04 + 1.21i$ for a detuning of -2.5 pm and an input power of 1 mW. Since $|\alpha^{FCA}| > 1$, we can no longer write $e^{\alpha_i^{FCA}\sin(\theta)} \sim I_0(\alpha_i^{FCA}) - iI_1(\alpha_i^{FCA})(e^{i\theta} - e^{-i\theta})$, $I_0(\alpha_i^{FCA}) \sim 1$ nor $I_1(\alpha_i^{FCA}) \sim \frac{1}{2}\alpha_i^{FCA}$, and the formulae above are no longer very accurate. However, they can still be used to estimate the importance of the second harmonic. Numerically, for a detuning of -2.5 pm and an input power P_{in}= 1 mW, similar to the experimental parameters used in Ref. [16], $C_0 = 0.37$, $E_1/E_0 = -6$ dB and $E_2/E_1 \simeq -14.6$ dB according to the formula (37), a value which is close to the value given by a full numerical simulation (- 15 dB). Therefore, we can conclude that the greater nonlinearity of the oscillations in a microring resonator is caused firstly by the higher value of C_0, itself explained by the lower value of the frequency of the oscillations in a microdisk, and secondly by the higher amplitude of those oscillations.

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